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## Symmetry of superconductivity in fullerenes by repulsive interaction model

S. Yamazaki\*, Y. Kuramoto

Tohoku University, 6-3 Aoba, Aoba-ku, Sendai 980-8578, Japan

### Abstract

A repulsive interaction model for superconductivity (SC) in fullerenes is studied using a second-order perturbation theory with respect to Coulomb repulsion. With the purely repulsive interaction,  $T_g$  and  $E_g$  pairs are the most stable in bcc and fcc lattices, respectively. In addition to anisotropic SC pairs, the s-wave ( $A_g$ ) pair can also be formed in both lattice models. Multiple Fermi surfaces in fullerenes stabilize the  $A_g$  pairs. We propose that the cooperation between Coulomb repulsion and electron-phonon interaction is responsible for the high  $T_c$ .

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### 1. Introduction

The mechanism of the high  $T_c$  is discussed by many researchers. Alkali metal doped fullerenes ( $A_3C_{60}$ ) provide an interesting case for such study.  $A_3C_{60}$  shows not only superconductivity with high  $T_c$  up to about 40K, but also antiferromagnetism with  $A=Cs$ . Namely,  $Cs_3C_{60}$  is a Mott insulator at ambient pressure [1], while other  $A_3C_{60}$  with  $A = K, Rb$  are superconductors at low temperature. The insulating state of  $Cs_3C_{60}$  changes to superconducting (SC) state under applied pressure, which is in line with other organic superconductor [2]. In view of nearby presence of the metal-insulator transition,  $A_3C_{60}$  is a strongly correlated system. It is known that fullerenes take two different cubic structures; A15 (bcc like) and fcc lattices [3]. The conduction bands of  $A_3C_{60}$  are composed of three-fold degenerate  $t_{1u}$  molecular orbitals of a  $C_{60}$  [4]. The SC states with orbital degenerate system are attracting recent interest since the discovery of iron pnictide compounds [5].

It has been discussed that the SC state of  $A_3C_{60}$  is driven mainly by the electron-phonon interaction. In the conventional theory, the s-wave state is unfavorable in the presence of strong Coulomb repulsion. Han *et al* have pointed out that the dynamic Jahn-Teller effect is responsible for reducing the strong Coulomb repulsion, and thereby stabilizing the ordinary s-wave state [6]. The fundamental question, however, remains why the  $T_c$  is so high, since the effective Coulomb repulsion should remain significant even in the SC state.

The interplay of the Coulomb and electron-phonon interactions is a highly delicate matter in the strong-coupling region with high  $T_c$ . As a possible step toward the complete understanding, we study purely repulsive models with the characteristic band structure. We discuss here which symmetry is most stabilized in the presence of degenerate orbitals and three dimensional lattice structures.

### 2. Formalism

To discuss the difference between bcc and fcc lattices, we consider tight binding models which are derived from the three degenerate molecular orbitals of each  $C_{60}$ . The hopping integrals are fitted by first principle calculations for

\*Corresponding author. Tel.: +81-22-795-6444; fax: +81-22-795-6447.  
E-mail address: [yamazaki@cmpt.phys.tohoku.ac.jp](mailto:yamazaki@cmpt.phys.tohoku.ac.jp).

each lattice model [4, 7]. In this article, we focus on the on-site Coulomb repulsion as a pairing interaction. Taking the second-order perturbation theory according to Ref. [9], we derive the Kohn-Luttinger (KL) interaction for  $A_3C_{60}$  as follows [8-10]:

$$V_{mn,m'n'}(k, k') = [\widehat{U} + \widehat{U} \widehat{\chi}(k + k) \widehat{U}]_{mn,m'n'}, \quad (1)$$

$$\chi_{mn,m'n'}(q) = -\frac{1}{N} \sum_{p,l,l'} \bar{A}_l^{(m)}(p+q) \bar{A}_l^{(m')}(p+q) A_{l'}^{(n)}(p) \bar{A}_{l'}^{(n')}(p) \frac{f(\varepsilon_l(p+q)) - f(\varepsilon_{l'}(p))}{\varepsilon_l(p+q) - \varepsilon_{l'}(p)}, \quad (2)$$

where  $m, n$  denote orbital indices, and  $\widehat{U}$  is the Coulomb interaction matrix which includes intra-orbital Coulomb  $U$ , inter-orbital Coulomb  $U'$ , the Hund's coupling  $J$ , and the pair hopping  $J'$ . Here we neglect  $J$  and  $J'$  and impose the relation  $U' = U$  for simplicity. Then the matrix elements exist only for the components:  $\widehat{U}_{mn,mn} = U$  ( $m=n$ ). The other components are set to zero. The matrix  $\widehat{\chi}$  is the momentum-dependent static susceptibility, and  $\varepsilon_l(p)$  is the energy with  $l$  band given by our tight binding model.  $A_l^{(m)}(p)$  is a weight of orbital ( $m$ ) basis in  $l$  band.  $f(\varepsilon) = 1/(e^{\beta(\varepsilon-\mu)} + 1)$  is the Fermi distribution function with  $\mu$  is the chemical potential and  $\beta$  the inverse of temperature.  $N$  is number of lattice points in the system. In order to investigate the SC instability toward the SC state in  $A_3C_{60}$ , we solve the following eigenvalue equations [11, 12]:

$$\lambda \Delta_l(k) = -\frac{1}{(2\pi)^3} \sum_{k'l'} \frac{dS_{k'l'}}{v_F(k',l')} V_{ll'}(k, k') \Delta_{l'}(k'), \quad (3)$$

$$V_{ll'}(k, k') = \sum_{mn,mn} \bar{A}_l^{(m)}(k) \bar{A}_l^{(n)}(-k) V_{mn,m'n'}(k, k') A_{l'}^{(m')}(k') \bar{A}_{l'}^{(n')}(-k'). \quad (4)$$

where  $\Delta_l(k)$  is the SC gap function with band  $l$ ,  $dS_{k,l}$  denotes a Fermi surface (FS) element of the band  $l$  with  $v_F$  being the Fermi velocity. In the following, we discuss the largest  $\lambda$  which gives the maximum  $T_c$ . Since  $T_c$  is much smaller than the characteristic energy of  $\widehat{\chi}$  in weak coupling region, we put  $T=0$  in Eq. (2). We evaluate the surface integral by using the tetrahedron method [13], and the eigenvalue equation is solved by the power method. In our calculation, more than  $10^4$  elements on FS are used. Our approach is accurate only for  $U$  much smaller than the bandwidth  $W$ .

### 3. Numerical result

We have derived the maximum  $\lambda$  for each symmetry with bcc and fcc crystal structures. In the following, we consider only the case of three conduction electrons per  $C_{60}$ . The lattice constant is set to unity. We fix the band width  $W$  to 0.5 eV. Then we obtain the simple scaling  $\lambda \propto U^2$ , provided only the  $O(U^2)$  term in Eq. (2) is active. The values of  $\lambda$  computed for each symmetry are summarized in Table 1.

Table 1. The SC coupling constant  $\lambda/U^2$  for each irreducible representation. The unit of  $U$  is eV, which means  $\lambda$  takes the numerical value in the table with  $U=1$  eV.  $A_g, E_g$  and  $T_g$  pairs are spin singlet states while  $T_u$  pair is a spin triplet state.

Crystal structure	$A_g$	$E_g$	$T_g$	$T_u$
Bcc lattice	0.2358	0.1349	0.6287	0.2860
Fcc lattice	0.2016	0.2926	0.2552	0.1869

Table 1 shows the  $T_g$  symmetry of the pair is the most favoured in the bcc model. The eigenstate has the three-fold degeneracy with components ( $d_{xy}, d_{yz}, d_{zx}$ ). The stability of the  $T_g$  pairing is related to the shape of Fermi surfaces and a nesting vector  $Q_H$ . We show in Fig.1(a) the gap function  $\Delta_l(k)$  of the  $d_{xy}$  state along the FS in the  $k_z=0$  plane in Brillouin zone (BZ). The numerical values shown by color mean the amplitude with normalization over the whole FS. The red and blue parts represent positive and negative  $\Delta_l(k)$ , respectively. The boundary between red part and blue part becomes a node. It is known in the single band bcc model [12] that the SC state with  $T_g$  symmetry is most stabilized. With the  $d_{xy}$  symmetry, there is substantial combination of momenta that satisfy  $k + k' \cong Q_H$ . Hence, the effective interaction  $V_{ll'}(k, k')$  becomes most effective for the  $d_{xy}$  state. Similar situation occurs to other members  $d_{yz}, d_{zx}$  of the  $T_g$  symmetry. Note that the gap functions of all bands have the same sign for each quadrant of BZ in Fig. 1. This is because each band has comparable size of FS, and compose almost degenerate bands.

On the other hand, Table 1 shows that the SC state with  $E_g$  symmetry is the most stable in the fcc model, but the difference with other symmetry is not large. In contrast with the  $O_h$  group, which has  $d_{x^2-y^2}$  and  $d_{3z^2-r^2}$  states as eigenfunctions, the  $T_h$  symmetry have no four-fold symmetry. In our result,  $\Delta_l(k)$  corresponding to band 1 is close to

$d_{x^2-y^2}$  state, while  $\Delta_2(k)$  is rather different from the  $E_g$  state in the  $O_h$  group. The node structure of  $\Delta_l(k)$  in  $E_g$  symmetry is complicated.

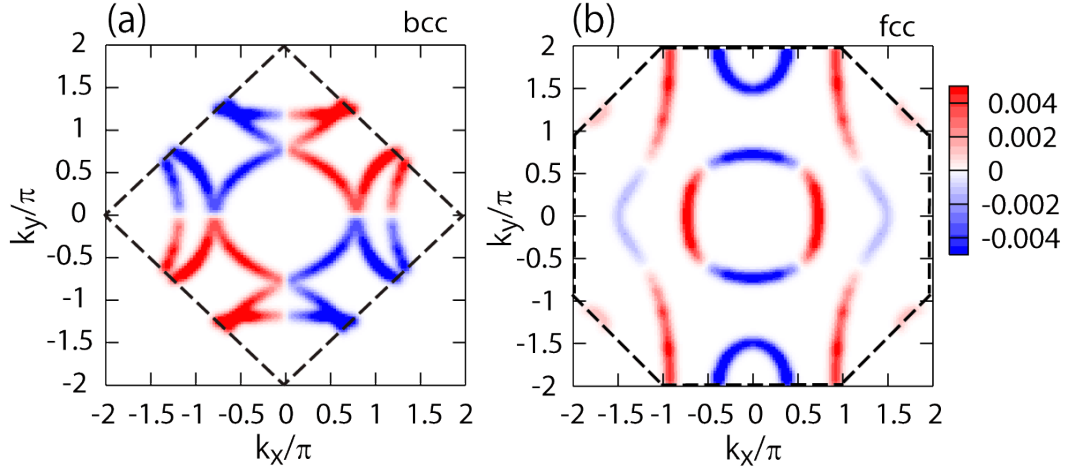


Fig. 1. Illustration of the gap function for the most stable symmetry for each lattice structure, (a) bcc model; (b) fcc model. The gap function is projected onto the plane  $k_z=0$ . The dashed lines show the boundary of BZ. The gap function of fcc and bcc models have  $T_g$  and  $E_g$ , respectively.

We next move to discuss the pairing with the  $A_g$  symmetry which is less stable in the present model, but which seems most relevant to actual fullerenes. Figure 2 shows the gap function  $\Delta_l(k)$  with the  $A_g$  symmetry for each structure. Different signs of the gap functions work to cancel the  $O(U)$  part of the KL interaction. We emphasize such cancellation requires nodes along the FS in the single band model. In our case, bcc model has nodes as in the single band model, while fcc model cancel the  $O(U)$  without nodes, and hence having less cost in the kinetic energy. In Table 1, the coupling constant  $\lambda$  with  $A_g$  symmetry in bcc model is slightly larger than the fcc model.

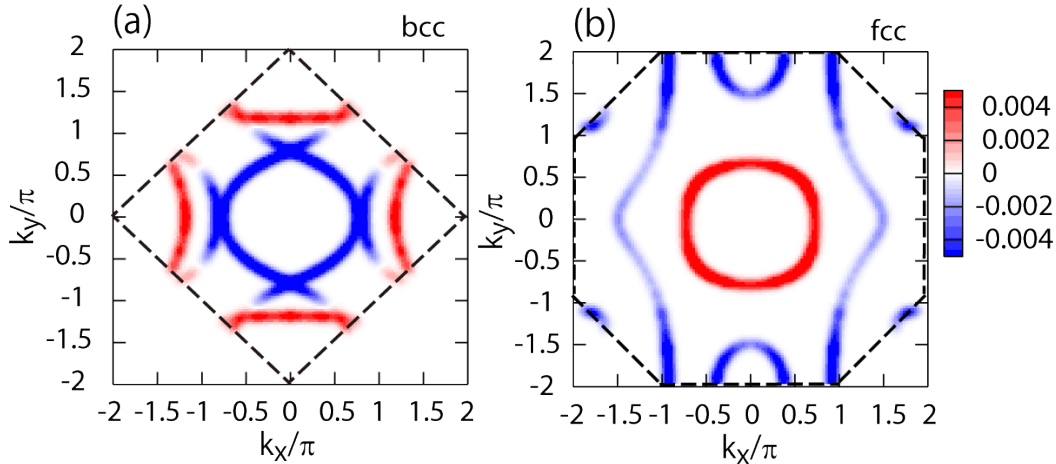


Fig. 2. Illustration of the gap function with  $A_g$  symmetry in the  $k_z=0$  plane in each model. (a) In the bcc model, the gap function has nodes keeping the  $A_g$  symmetry. (b) In the fcc model, the two sheets of FS have opposite signs of  $\Delta_l(k)$ .

#### 4. Discussion

We have studied the repulsive interaction model for superconductivity in  $A_3C_{60}$ . By using the second order perturbation theory with respect to the Coulomb repulsion, we calculate the SC coupling constant  $\lambda$  for various SC symmetries and for both bcc and fcc lattices. By the nesting property of the FS in the bcc model, the  $T_g$  symmetry is the most favorable within the repulsive interaction model. An anisotropic SC state is the most stable also in the fcc model. In the fcc model, various SC symmetries have comparable magnitudes for coupling constant  $\lambda$ . We have further shown that the stable pair with the fully symmetric  $A_g$  can also be formed by purely repulsive interaction. We emphasize that the presence of multiple conduction bands is essential for the stability of  $A_g$  pair.

Let us now discuss possible relevance of our results to actual superconductivity in fullerenes, especially in  $Cs_3C_{60}$  with the highest  $T_c$ . The most fundamental question is why the  $T_c$  is so high. Since the system is close to the Mott transition, one might naively guess that the SC is caused by a non-phonon mechanism as in cuprate superconductors. However, various experimental evidences exclude the presence of an anisotropic SC state, like d-wave pairing. According to our results, we propose that a following scenario for the SC in  $Cs_3C_{60}$ : What is responsible for the high  $T_c$  is the cooperation, rather than competition, between the Coulomb repulsion and the Jahn-Teller phonons. The  $A_g$  pair will be most favorable for such cooperation. However, it is not clear whether the nodes remain in the presence of Jahn-Teller phonons. If the nodes remain, a gapless SC state is realized. With purely repulsive interaction, the  $A_g$  state in the bcc model has nodal lines while there is a full gap in the fcc model. Theoretically, simultaneous account of Jahn-Teller phonons together with Coulomb repulsion requires a new scheme which is not available at present. We have of course recognized that the dynamical mean field theory (DMFT) can powerfully deal with local Coulomb correlation together with Jahn-Teller phonons [6]. The KL interaction is highly non-local and hence is beyond the scope of the DMFT. Various available schemes in the momentum space, on the other hand, are not reliable enough to deal with the case of strong Coulomb repulsion. Namely, SC in fullerenes provides a challenging ground to construct a new theoretical scheme.

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